Statistics
Lecture 11

Feb 19-8:47 AM

SG 22

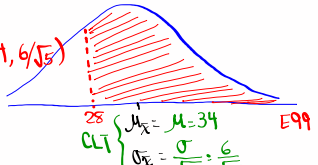
Ages of College students are normally dist with mean of 34 Yrs and standard deviation of 6 Years. $N(34, 6)$

If we randomly Select $n = 5$ Students, find the prob. that their mean ($\bar{x}$) age is more than 28 Yrs. (> 28)

$P(\bar{x} > 28)$

= normalcdf(28, E99, 34, $6/\sqrt{5}$)

= .987 ≈ 98.7%



CLT $\mu_{\bar{x}} = \mu = 34$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{5}}$

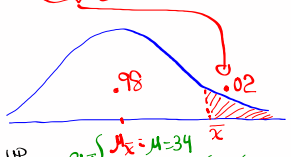
$n = 4$

For randomly Selected groups of 4, find the $\bar{x}$ mean that separates the top 2% from the rest. Round to a whole #.

$\bar{x}$ = invNorm(.98, 34, 3)

= 40.16 Y

≈ 40 → Round-up 41



CLT $\mu_{\bar{x}} = \mu = 34$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = \frac{6}{2} = 3$

Nov 7-6:51 PM

SG 22 & SG 23

Estimating Parameters

Statistics → Sample

Parameters → Population

we use statistic to estimate parameters

Estimation of a parameter will be in the form of range of values.

↳ Confidence Interval

Building a Confidence Interval comes with some level of confidence

Common C-level: 90%, 95%, 99%

Notation: Confidence level is $(1-\alpha)\cdot 100\%$

where $0 < \alpha < 1$ and is called Significance level.

↑ Alpha

So if $\alpha = .1$ → Sig. level. → $(1-\alpha)\cdot 100\% = (1-.1)\cdot 100\% = 90\%$ ↑ Conf. level

If $\alpha = .05$ → $(1-\alpha)\cdot 100\% = (1-.05)\cdot 100\% = 95\%$

Your turn

If $\alpha = .02$ → $(1-\alpha)\cdot 100\% = (1-.02)\cdot 100\% = 98\%$

If α not given → use .05

If C-level not given → use .95

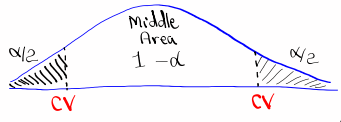
Nov 7-7:03 PM

What is α? what does it represent?

$0 < \alpha < 1$, Significance level

$(1-\alpha)\cdot 100\%$ Confidence level

think of $1-\alpha$ as the middle area of the graph of Prob. dist. and $\alpha/2$ is the area on each tail.

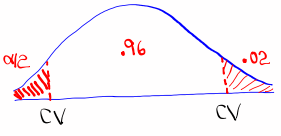


Values that Separate the tail areas from the middle area are Called Critical Values (CV).

Ex. $\alpha = .04$

$1-\alpha = .96$

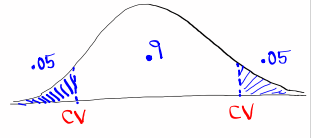
$\alpha/2 = .02$



$\alpha = .1$

$1-\alpha = .9$

$\alpha/2 = .05$



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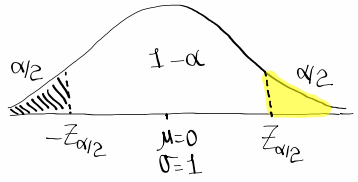
$z_{\alpha/2}$ is a critical value for standard normal prob. dist. with right-tail area of $\alpha/2$.

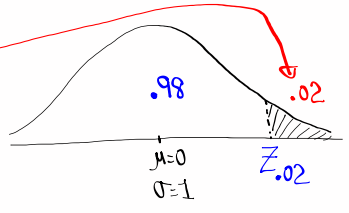


How to find $z_{\alpha/2}$:

Use invNorm(Left area, 0, 1)

Nov 7-7:23 PM

Find $z_{.02}$



$\frac{\alpha}{2} = .02$

$\alpha = .04$

$1 - \alpha = .96$

$z_{.02} = \text{invNorm}(.98, 0, 1)$

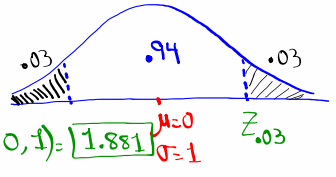
$= \boxed{2.054}$

Find $z_{\alpha/2}$ for 94% C-level.

Middle area = .94

$1 - .94 = .06$

$.06 \div 2 = .03$



$z_{.03} = \text{invNorm}(.97, 0, 1) = \boxed{1.881}$

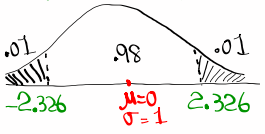
Nov 7-7:26 PM

Find $z_{.01}$

$z_{.01}$ = invNorm(.99, 0, 1)

= 2.326

98% C-level



Nov 7-7:32 PM

Estimating Population Proportion

Final Ans: $\boxed{\quad < p <}$ Confidence Interval

$\hat{p}$

$\hat{p} - E < p < \hat{p} + E$

$\hat{p}$: Sample Proportion, "Point-estimate"

E: Margin of error

for example, $\hat{p} = .2$ and $E = .04$

$\hat{p} - E < p < \hat{p} + E$

$.2 - .04 < p < .2 + .04$

$\boxed{.16 < p < .24}$

Nov 7-7:36 PM

In a sample of 400 voters, 175 of them were in support of abortion rights.

$\hat{p} = \frac{x}{n} = \frac{175}{400} = .438$

with margin of error of 5%,

$\hat{p} - E < p < \hat{p} + E$

$.438 - .05 < p < .438 + .05$

$\boxed{.388 < p < .488}$ Conf. interval for Prop. of all voters in support of abortion rights.

Nov 7-7:41 PM

How to find margin of error E:

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p}$ Sample Prop.

$\hat{q} = 1 - \hat{p}$

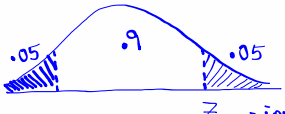
n Sample Size

$z_{\alpha/2}$ CV for $(1-\alpha)\cdot 100\%$ C-level.

Suppose $n = 100$, $p = .8$, C-level $= 90\%$.

Find E.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \sqrt{\frac{(.8)(.2)}{100}} = \boxed{.07}$



$z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

$\hat{p} - E < p < \hat{p} + E$

$.8 - .07 < p < .8 + .07$

$\boxed{.73 < p < .87}$

We are 90% confident that Pop. Prop. falls between 73% & 87%.

Nov 7-7:46 PM

In a survey of 300 people, 80% of them had iPhone. ($n=300$, $\hat{p}=.8$)

1) How many of them had iPhone.

$\hat{p}=\frac{x}{n}$ $x=n\hat{p}=300(.8)=240$

if decimal, Round-up

2) Construct 90% Conf. interval for the prop. of all people that have iPhone.

C-level: .9

STAT → TESTS ↓ 1-PropZInt

$x=240$
$n=300$
C-level: .9
Calculate

$.762<p<.838$

$.76<p<.84$

We are 90% Confident that between 76% and 84% of all people have iPhone.

3) Find the margin of error.

$E=\frac{.84-.76}{2}=.04$

$\hat{p}=\frac{.84+.76}{2}=.8$

Nov 7-8:06 PM

I Surveyed 185 students, and 42% of them were in support of online classes.

$n=185$, $\hat{p}=.42$ → $x=n\hat{p}=185(.42)\approx 78$

If decimal → Round-up

Construct Conf. interval for the prop. of all Students in Support of online classes.

No C-level
⇒ use .95

$.350<p<.493$

1-PropZInt
$x=78$
$n=185$
C-level: .95
Calculate

$.35<p<.49$

We are 95% Confident that between 35% & 49% of all students are in support of online classes.

$E=\frac{.49-.35}{2}=.07$

$\hat{p}=\frac{.49+.35}{2}=.42$

Nov 7-8:17 PM

Estimating Population mean: μ

Final Ans: $< \mu <$

Conf. Interval $\bar{x} - E < \mu < \bar{x} + E$ (Margin of error)

$\bar{x}$: Sample Mean "Point-estimate"

Case I: σ Known

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

TI Command

ZInterval

input: Stats

Nov 7-8:26 PM

Given $n = 32$, $\bar{x} = 80$, $\sigma = 10$, C-level: .98

Find C.I. for μ.

Since σ is Known → use ZInterval

STAT → TESTS ↓ ZInterval

$E = \frac{84 - 76}{2} = 4$

$\bar{x} = \frac{84 + 76}{2} = 80$

Input: Stats

$\sigma = 10$

$\bar{x} = 80$

$n = 32$

C-level: .98

Calculate

$75.888 < \mu < 84.112$

whole

$76 < \mu < 84$

Nov 7-8:31 PM

I randomly selected 28 exams, the mean score was 83.5.
$n = 28$
$\bar{x} = 83.5$
It is known that Standard deviation of all exams is 10.2.
$\sigma = 10.2$

Find 99% Conf. interval for the mean score of all exams. C-level: .99

Since σ is known we use ZInterval
inpt: Stats
$\sigma = 10.2$
$\bar{x} = 83.5$
$n = 28$
C-level: .99
Calculate

$E = \frac{88.5 - 78.5}{2} = 5$

$\bar{x} = \frac{88.5 + 78.5}{2} = 83.5$

$78.535 < \mu < 88.465$

$78.5 < \mu < 88.5$

1-decimal

I am 99% Confident the mean of all Scores falls between 78.5 & 88.5.

Nov 7-8:37 PM

Estimating Population mean: μ

Final Ans: $\_ < \mu < \_$

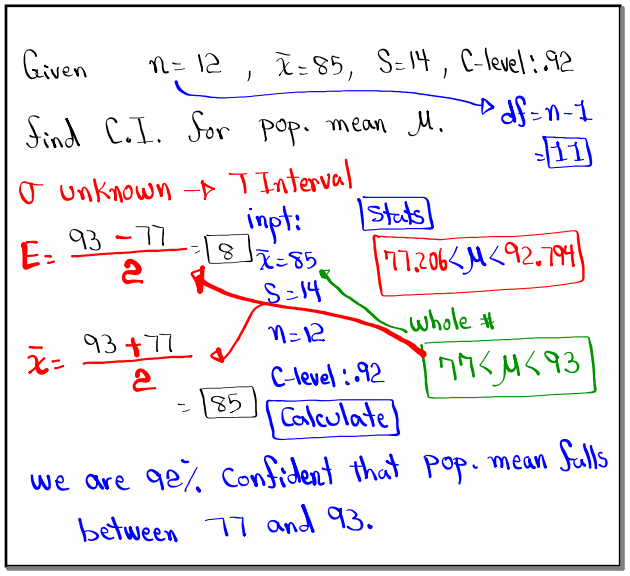
Conf. Interval $\bar{x} - E < \mu < \bar{x} + E$

Margin of error

$\bar{x}$: Sample Mean, "Point-estimate"

Case I: σ Known	Case II: σ Unknown
$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$, $df = n-1$
TI Command: ZInterval inpt: Stats	TI Command: TInterval inpt: STATS

Nov 7-8:26 PM



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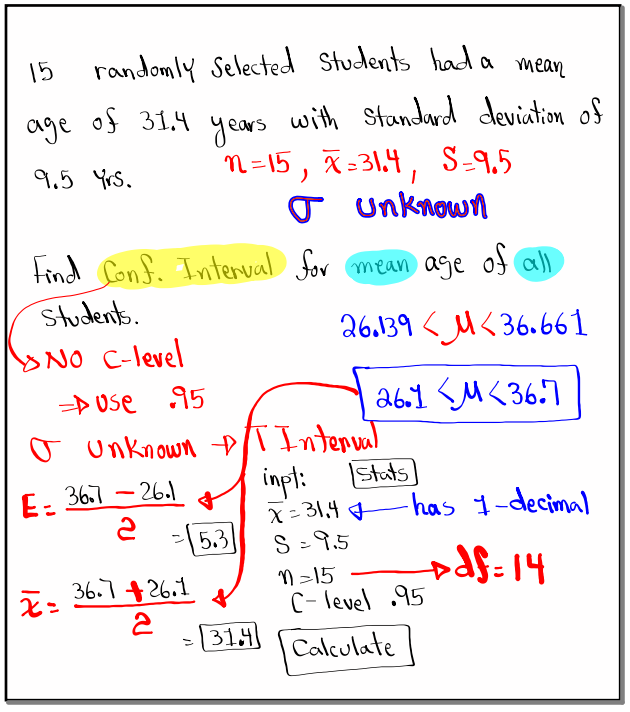

Nov 7-8:56 PM

I randomly Selected 12 exams. Here are the Scores.

75	82	98	100
68	55	72	90
58	77	90	95

1) Find $\bar{x}$ & S.
Round to whole #.
$\bar{x}=80$ $S=15$

2) Find 99% C.I. for the mean of all exams.

σ unknown → T Interval
Inpt: Stats
$\bar{x}=80$
$S=15$
$n=12$
C-level: .99
Calculate

$66.551 < \mu < 93.449$
$67 < \mu < 93$
whole #

df = 11

$E = \frac{93-67}{2} = 13$

$\bar{x} = \frac{93+67}{2} = 80$

Exam 3: Nov. 18, 2023

Nov 7-9:06 PM